Exercise 40

(a) If $f(x) = (x^2 - 1)e^x$, find $f'(x)$ and $f''(x)$.

(b) Check to see that your answers to part (a) are reasonable by comparing the graphs of f, f', and f''.

Solution

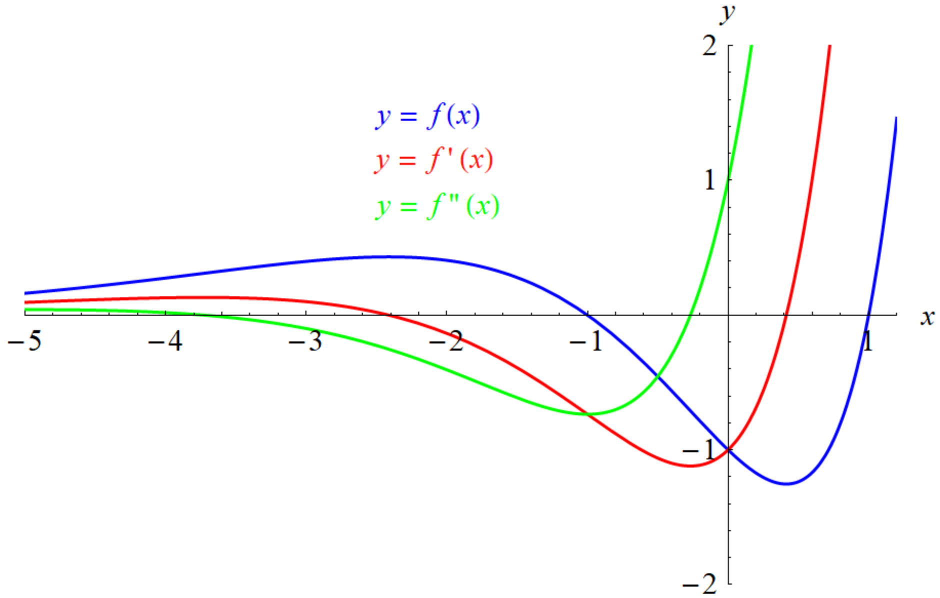
Evaluate the derivative using the product rule.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[(x^2 - 1)e^x\right] \\ &= \left[\frac{d}{dx}(x^2 - 1)\right](e^x) + (x^2 - 1)\left[\frac{d}{dx}(e^x)\right] \\ &= (2x)(e^x) + (x^2 - 1)(e^x) \\ &= (x^2 + 2x - 1)e^x \end{aligned}$$

Evaluate the second derivative using the product rule again.

$$\begin{aligned} f''(x) &= \frac{d}{dx}[f'(x)] \\ &= \frac{d}{dx}\left[(x^2 + 2x - 1)e^x\right] \\ &= \left[\frac{d}{dx}(x^2 + 2x - 1)\right](e^x) + (x^2 + 2x - 1)\left[\frac{d}{dx}(e^x)\right] \\ &= (2x + 2)(e^x) + (x^2 + 2x - 1)(e^x) \\ &= (x^2 + 4x + 1)e^x \end{aligned}$$

Below is a graph of the function and its derivatives versus x.



$f'(x)$ is positive wherever $f(x)$ increases, $f'(x)$ is zero wherever the slope of $f(x)$ is zero, and $f'(x)$ is negative wherever $f(x)$ is decreasing.

Similarly, $f''(x)$ is positive wherever $f'(x)$ increases, $f''(x)$ is zero wherever the slope of $f'(x)$ is zero, and $f''(x)$ is negative wherever $f'(x)$ is decreasing. The answers in part (a) are reasonable then.